Aufgaben zur partiellen Integration

E Berechnen Sie das unbestimmte Integral zu folgenden Funktionen $f(x)$,

	geg.: $f(x)$	ges.: $\int f(x)dx$		geg.: $f(x)$	ges.: $\int f(x)dx$
$E1$	$x \cos x$		$E2$	$x^2 \sin x$	
$E3$	$x\, e^{-x/a}$		$E4$	$(x^2 + 7x - 5) \cos 2x$	
$E5$	$x \sin(ax + b)$		$E6$	$x \sqrt{x + a}$	
$E7$	$x^n \ln \lvert x \rvert$		$E8$	$\frac{\ln x}{x}$	

(1)

$$\int u \cdot v' \, dx = u \cdot v - \int u' \cdot v \, dx$$

E1: $\int x \cos x dx$	$= x \sin x - \int \sin x dx$ $= x \sin x + \cos x + C$	$u = x,\ v' = \cos x,\ u' = 1,\ v = \sin x$
E2: $\int x^2 \sin x dx$	$= -x^2 \cos x + 2 \int x \cos x dx$ $= -x^2 \cos x + 2x \sin x + 2 \cos x + C$	$u = x^2,\ v' = \sin x,\ u' = 2x,\ v = -\cos x$ siehe E1
E3: $\int x\, e^{-x/a} dx$	$= -ax\, e^{-x/a} + a \int e^{-x/a} dx$ $= -ax\, e^{-x/a} - a^2 e^{-x/a} + C$	$u = x,\ v' = e^{-x/a},\ u' = 1,\ v = -ae^{-x/a}$ $\int e^{-x/a} dx = -a\, e^{-x/a} + C$

$$\int u \cdot v' \, dx = u \cdot v - \int u' \cdot v \, dx$$

E4: $\int (x^2 + 7x - 5) \cos 2x \, dx$ $\qquad u = x^2 + 7x - 5 \quad v' = \cos 2x,$

$$= \frac{1}{2}(x^2 + 7x - 5) \sin 2x - \frac{1}{2} \int (2x + 7) \sin 2x \, dx \qquad u' = 2x + 7 \qquad v = \frac{1}{2} \sin 2x$$

Nebenrechnung: $\qquad u = 2x + 7 \qquad v' = \sin 2x,$

$$\int (2x+7) \sin 2x \, dx = -\frac{1}{2}(2x+7) \cos 2x + \int \cos 2x \, dx \qquad u' = 2 \qquad v = -\frac{1}{2} \cos 2x$$

$$= \frac{1}{2}(x^2+7x-5) \sin 2x + \frac{1}{4}(2x+7) \cos 2x - \frac{1}{2} \int \cos 2x dx$$

$$= \frac{1}{2}(x^2+7x-5) \sin 2x + \frac{1}{4}(2x+7) \cos 2x - \frac{1}{4} \sin 2x + C$$

$$\int u \cdot v' \, dx = u \cdot v - \int u' \cdot v \, dx$$

E5: $\int x \sin(ax+b)\, dx$	$u = x,\ v' = \sin(ax+b),$
$= \quad -\frac{x}{a} \cos(ax+b) + \frac{1}{a} \int \cos(ax+b)\, dx$	$u' = 1,\ v = -\frac{1}{a} \cos(ax+b)$
$= \quad -\frac{x}{a} \cos(ax+b) + \frac{1}{a^2} \sin(ax+b) + C$	
E6: $\int x \sqrt{x+a}\, dx =$	$u = x,\ v' = \sqrt{x+a}, u' = 1$
	$v = \int \sqrt{x+a}\, dx = \int (x+a)^{1/2} dx = \frac{2}{3}(x+a)^{3/2}$
$= \quad \frac{2x}{3}(x+a)^{3/2} - \frac{2}{3} \int (x+a)^{3/2} dx$	$\int (x+a)^{3/2} dx = \frac{2}{5}(x+a)^{5/2}$
$= \quad \frac{2x}{3}(x+a)^{3/2} - \frac{4}{15}(x+a)^{5/2} + C$	

$$\int u \cdot v' \, dx = u \cdot v - \int u' \cdot v \, dx$$

E7: $\int x^{n} \ln|x| dx$ $\qquad u = \ln|x|,\ v' = x^{n}$

$$= \frac{x^{n+1}}{n+1} \ln|x| - \int \frac{x^{n+1}}{(n+1)x} dx \qquad u' = \frac{1}{x},\ v = \frac{x^{n+1}}{n+1}$$

$$= \frac{x^{n+1}}{n+1} \ln|x| - \frac{1}{(n+1)} \int x^{n} dx$$

$$= \frac{x^{n+1}}{n+1} \ln|x| - \frac{x^{n+1}}{(n+1)^2} + C \tag{5}$$

E8: $\int \frac{\ln|x|}{x} dx$ $\qquad u = \ln|x|,\ v' = \frac{1}{x}$

$$= [\ln|x|]^2 - \int \frac{\ln|x|}{x} dx \qquad u' = \frac{1}{x},\ v = \ln|x|,$$

$$= \frac{1}{2} [\ln|x|]^2 + C$$